GCE AS/A level

0975/01

MATHEMATICS – C3

Pure Mathematics

A.M. WEDNESDAY, 22 January 2014

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. *(a)* Use Simpson's Rule with five ordinates to find an approximate value for the integral

$$\int_0^{\frac{\pi}{3}} \tan^2 x \, \mathrm{d}x.$$

Show your working and give your answer correct to four decimal places. [4]

(b) **Use your answer to part *(a)*** to deduce an approximate value for the integral

$$\int_0^{\frac{\pi}{3}} \sec^2 x \, \mathrm{d}x.$$ [2]

2. *(a)* Show, by counter-example, that the statement

'If x is an acute angle then $\sin(x + 30°) > \sin x$'

is false. [2]

(b) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$15\operatorname{cosec}^2\theta + 2\cot\theta = 23.$$ [6]

3. The curve C is defined by

$$x^3 - 2x^2y + 3y^2 = 3.$$

Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point $(-2, -1)$. [4]

4. The variables x and y are defined parametrically in terms of the variable t. It is known that $x = 2t^3$ and that $\frac{\mathrm{d}y}{\mathrm{d}x} = 2t + 4t^3$.

(a) Find an expression for $\frac{\mathrm{d}x}{\mathrm{d}t}$ in terms of t. [1]

(b) Find an expression for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ in terms of t and hence show there is no value of t for which

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2.$$ [4]

(c) Given that $y = 10$ when $t = 1$, find an expression for y in terms of t. [5]

5. **You may assume** that the equation $x^3 + 7x^2 - 3 = 0$ has a root α between 0 and 1.
The recurrence relation

$$x_{n+1} = \sqrt{\frac{3}{x_n + 7}}$$

with $x_0 = 1$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4.

Write down the value of x_4 correct to five decimal places and show this is the value of α correct to five decimal places. [5]

6. Differentiate each of the following with respect to x, simplifying your answer wherever possible.

(a) $(5x^3 - x)^{10}$ *(b)* $\sin^{-1}(x^3)$ [2], [2]

(c) $x^4 \ln(2x)$ *(d)* $\dfrac{e^{4x}}{(2x+3)^6}$ [3], [4]

7. *(a)* Find each of the following, simplifying your answer wherever possible.

(i) $\int e^{\frac{5x}{6}}\,dx$, (ii) $\int \sqrt[3]{8x+1}\,dx$, (iii) $\int \sin\left(1 - \frac{x}{3}\right)dx$. [6]

(b) Given that $a > 2$, and that

$$\int_2^a \frac{1}{4x-1}\,dx = 0{\cdot}284,$$

find the value of the constant a. Give your answer correct to one decimal place. [5]

8. Solve the equation

$$|3x + 4| = 2|x - 3|.$$ [3]

9. The function f has domain $[7, \infty)$ and is defined by

$$f(x) = 1 + \frac{2}{\sqrt{3x-5}}.$$

(a) Find an expression for $f^{-1}(x)$. [4]

(b) Write down the domain of f^{-1}. [2]

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10. The functions f and g have domains $(0, \infty)$ and $(-\infty, -2)$ respectively and are defined by

$$f(x) = \sqrt{x^2 + 5},$$
$$g(x) = \frac{-4}{x+1}.$$

(a) By considering $g'(x)$, show that g is an increasing function. [2]

(b) Write down the range of g. [2]

(c) Write down the domain and range of fg. [2]

(d) (i) Write down an expression for $fg(x)$.

(ii) Hence, solve the equation

$$fg(x) = 3.$$ [5]

END OF PAPER